Cheap Mathematics for MLE

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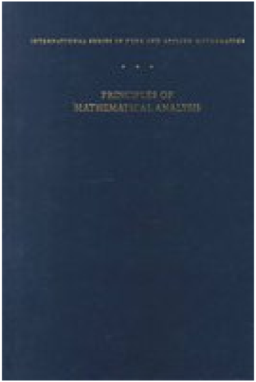
Rudinmania



Everything is in Rudin. Get it. Read it. Worship it.

Sups

- The *supremum* of a set S is the least upper bound.
- If $\sup$ of S exists, it does not need to be a member of S.
- Does $\sup$ exist for bounded subsets or $\Re$?

Sequences of functions

Let $\{f_n\}$ be a sequence of functions defined on a set E. Suppose that the sequence of real numbers

$$f_n(x) \to f(x)$$

for every $x \in E$. Then

$$\lim_{n\to\infty} f_n(x) = f(x), \qquad x \in E$$

Q: which properties of f_n are preserved by $f(x)$? (continuity, integrability, etc.),

Uniform convergence

$f_n(x)$ converges *uniformly* to $f(x)$ in E if for every $\epsilon > 0$, there is an integer N, such that

$$n \geq N \;\; \Rightarrow \;\; |f_n(x) - f(x)| \leq \epsilon,$$

for all $x \in E$.

Intuition? Eventually the whole sequence gets inside a single 'strip'

Cool properties

- If $f_n(x) \to f(x)$, then it does so uniformly iff

$$M_n \equiv \sup_{x \in E} |f_n(x) - f(x)| \longrightarrow 0$$

- The uniform limit of a sequence of continuous functions is continuous.

Typical example: $f_n(x) : [0,1] \to [0,1] : x^n$

Converges pointwisely but not uniformly. Each f_n is continuous but not the limit.

Mean values

Mean value theorem: If f is continuous on $[a, b]$ and differentiable in (a, b), $\exists\, x^* \in (a, b)$ at which

$$f(b) - f(a) = (b - a)\ f'(x^*)$$

- See picture
- At x^* the slope of the tangent and secant coincide.
- Use. Let 'free' $b = x$ and let it vary

$$f(x) = f(a) + f'(x^*)(x - a)$$

This is the **mean value approximation** of f at a. It is exact when $x \to a$, by definition of derivative.